DON BOSCO SCHOOL ALAKNANDA

CLASS XII (2017-18) **MATHEMATICS ASSIGNMENT : 1**

CHAPTER: INVERSE TRIGONOMETRIC FUNCTIONS

1. Write the principal value of:

   (i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (ii) $\sin^{-1}\left(-\frac{1}{2}\right)$ (iii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (iv) $\tan^{-1}\left(\sqrt{3}\right)$ (v) $\sec^{-1}(-2)$

   (vi) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cot^{-1}(0)$ (vii) $\tan^{-1}(1)-\cot^{-1}(-1)$

   (viii) $\sin^{-1}(-1)-\cos^{-1}(-1)+\tan^{-1}(-1)-\cot^{-1}(-1)+\sec^{-1}(-1)-\text{cosec}^{-1}(-1)$

2. Evaluate the following:

   (i) $\sin^{-1}(\sin(\frac{7\pi}{6}))$ (ii) $\tan^{-1}(\tan(\frac{11\pi}{6}))$

   (iii) $\cos\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{4}\right\}$ (iv) $\sin\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right\}$

   (v) $\tan(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4})$ (vi) $\cos(\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{3}{5})$

   (vii) $\sin^{-1}\left(-\frac{1}{2}\right)+2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (viii) $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)\right]$

   (ix) $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$ (x) $\tan^2(\sec^{-1}2)+\cot^2(\text{cosec}^{-1}3)$

   (xi) $2\left(\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)=\pi$ (xii) $\sin\left\{2\cot^{-1}\left(-\frac{5}{12}\right)\right\}$

   (xiii) $\sin^{-1}(\sin 4)$ (xiv) $\cos^{-1}(\cos 4)$

   (xv) $\sin^{-1}(\sin 2)$ (xvi) $\cos^{-1}(\cos 2)$

   (xvii) $\tan^{-1}(\tan 4)$ (xviii) $\tan^{-1}(\tan(-4))$

   (xix) $\tan(\tan^{-1}(-4))$ (xx) $\cos^{-1}(0.85)+\cos^{-1}(-0.85)$

3. Prove that $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\frac{14}{15}$
4. If $\tan^{-1}x=\frac{\pi}{10}$ for some $x\in\mathbb{R}$, then find the value of $\cot^{-1}x$.
5. Simplify and find the value of $y=\cos(2\cos^{-1}x+\sin^{-1}x)$ at $x=\frac{1}{5}$
6. Find the value of A if $A=\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$
7. Write in simplest form:

   (i) $\tan^{-1}(\frac{\cos x}{1-\sin x})$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$ (ii) $\tan^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right)$, $-a<x<a$

   (iii) $\tan^{-1}(x+\sqrt{1+x^2})$, $x\in R$ (iv) $\cot^{-1}\left(\frac{a}{\sqrt{x^2-a^2}}\right)$

   (v) $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$, $x\in\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]$ (vi) $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$

   (vii) $\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$ (viii) $\tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}$, $-a<x<a$

   (ix) $\sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$ (x) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

8. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, find the value of $\cot^{-1}x + \cot^{-1}y$
9. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}, x, y, z > 0$, find the value of xy +yz +zx
10. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find the value of x.
11. If $\sin(\cot^{-1}(x+1)) = \cos(\tan^{-1}x)$, then find the value of x.
12. Prove that x = ½ is a solution of $4\sin^{-1}x + \cos^{-1}x = \pi$
13. Prove that $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} = \frac{\pi}{4}$
14. Prove that:
    (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, xy<1.
    (ii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1+x^2}{1-x^2}\right)$
    (iii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$
15. Solve the following for x;
    (i) $\cot^{-1}2x + \cot^{-1}3x = \frac{\pi}{4}$
    (ii) $\tan^{-1}\left(\frac{-2x}{1-x^2}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \frac{2\pi}{3}$
    (iii) $\tan(\cos^{-1}x) = \sin(\tan^{-1}2)$; x>0.
    (iv) $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$
    (v) $\cot^{-1}x + \cot^{-1}(x+2) = \frac{\pi}{12}$
    (vi) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$
    (vii) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$
    (viii) $\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}x = 0$
16. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, then prove that a + b + c = abc
17. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$; prove that $x^2 + y^2 + z^2 + 2xyz = 1$.
18. If $a_1, a_2 \ldots a_n$ are in A.P. with common difference d, then simplify and evaluate
    $\tan\left(\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right)$
19. Prove the following:
    (i) $\cot\left[\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right] + \cos^{-1}(1-2x^2) + \cos^{-1}(2x^2-1) = \pi, x > 0.$
    (ii) $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$
    (iii) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$
    (iv) $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) - \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{\pi}{4}$
    (v) $\cot^{-1}\left(2\tan\left(\cos^{-1}\frac{8}{17}\right)\right) + \tan^{-1}\left(2\tan\left(\sin^{-1}\frac{8}{17}\right)\right) = \tan^{-1}\left(\frac{300}{161}\right)$
    (vi) $\tan^{-1}\left(\frac{m}{n}\right) - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$
    (vii) $\tan\left[\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right] = \frac{2\sqrt{a^2+b^2}}{b}$
    (viii) $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = \tan^{-1}\left(\frac{a}{b}\right) - x$
    (ix) $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\,tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right)$
    (x) $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{x+1}{\sqrt{2+2x+x^2}}\right) = \tan^{-1}(x^2 + x + 1)$
20. Find the domain and range of $y = \sin^{-1}(2x)$.