

Don Bosco School

Alaknanda, New Delhi 110019



Assignment Booklet

MATHEMATICS

CLASS XI

2017-2018

‘Things should be made as simple as possible but not any simpler’

– Albert Einstein (1879 -1955)

PREFACE

The syllabus in the subject of mathematics has undergone changes from time to time in accordance with the growth of the subject and emerging needs of the society. Secondary stage is a launching stage from where the students go either for higher academic education in mathematics or for professional courses like engineering, physical and biosciences, accounting , economics or computer applications.

‘Assignment 2017-‘18 is an effort to meet the emerging needs of all categories of students, using the topics from real life situations. The newly introduced value bases questions are incorporated to promote national integration, protection of environment, cultural values and self discipline.

We would like to thank our Principal, **Fr. Babu Varghese, SDB** for his constant encouragement and invaluable guidance in the teaching of mathematics.

All suggestions for further improvement of this booklet shall be thankfully acknowledged.

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Basic Math symbols

Symbol	Symbol Name	Meaning / definition	Example
=	equals sign	equality	$5 = 2+3$
\neq	not equal sign	inequality	$5 \neq 4$
>	strict inequality	greater than	$5 > 4$
<	strict inequality	less than	$4 < 5$
\geq	inequality	greater than or equal to	$5 \geq 4$
\leq	inequality	less than or equal to	$4 \leq 5$
()	parentheses	calculate expression inside first	$2 \times (3+5) = 16$
[]	brackets	calculate expression inside first	$[(1+2)*(1+5)] = 18$
\pm	plus - minus	both plus and minus operations	$3 \pm 5 = 8$ and -2
\mp	minus - plus	both minus and plus operations	$3 \mp 5 = -2$ and 8
*	asterisk	multiplication	$2 * 3 = 6$
\times	times sign	multiplication	$2 \times 3 = 6$
·	multiplication dot	multiplication	$2 \cdot 3 = 6$
\div	division sign / obelus	division	$6 \div 2 = 3$
/	division slash	division	$6 / 2 = 3$
—	horizontal line	division / fraction	$\frac{6}{2} = 3$
mod	modulo	remainder calculation	$7 \bmod 2 = 1$
a^b	power	exponent	$2^3 = 8$
a^b	caret	exponent	$2 \wedge 3 = 8$
\sqrt{a}	square root	$\sqrt{a} = a^{\frac{1}{2}}$	$\sqrt{9} = \pm 3$
$\sqrt[3]{a}$	cube root	$\sqrt[3]{a} = a^{\frac{1}{3}}$	$\sqrt[3]{8} = 2$
$\sqrt[n]{a}$	n-th root (radical)	$a^{\frac{1}{n}}$	for $n=3$, $\sqrt[n]{8} = 2$
%	percent	$1\% = 1/100$	$10\% \times 30 = 3$
‰	per-mille	$1\text{‰} = 1/1000 = 0.1\%$	$10\text{‰} \times 30 = 0.3$

Geometry symbols

Symbol	Symbol Name	Meaning / definition	Example
\sphericalangle	angle	formed by two rays	$\sphericalangle ABC = 30^\circ$
\perp	right angle	$= 90^\circ$	$\alpha = 90^\circ$
$^\circ$	degree	1 turn = 360°	$\alpha = 60^\circ$
$'$	Arc minute	$1^\circ = 60'$	$\alpha = 60^\circ 59'$
$''$	Arc second	$1' = 60''$	$\alpha = 60^\circ 59' 59''$
\widehat{AB}	arc	arc from point A to point B	
\perp	perpendicular	perpendicular lines (90° angle)	$AC \perp BC$
\parallel	parallel	parallel lines	$AB \parallel CD$
\cong	congruent to	equivalence of geometric shapes and size	$\triangle ABC \cong \triangle XYZ$
\sim	similarity	same shapes, not same size	$\triangle ABC \sim \triangle XYZ$
\triangle	triangle	triangle shape	$\triangle ABC \cong \triangle BCD$
$ x-y $	distance	distance between points x and y	$ x-y = 5$
π	pi constant	$\pi = 3.141592654\dots$ is the ratio between the circumference and diameter of a circle	$c = \pi \cdot d = 2 \cdot \pi \cdot r$
rad	radians	radians angle unit	$360^\circ = 2\pi \text{ rad}$
grad	grads	grads angle unit	$360^\circ = 400 \text{ grad}$

Algebra symbols

Symbol	Symbol Name	Meaning / definition	Example
x	x variable	unknown value to find	when $2x = 4$, then $x = 2$
\equiv	equivalence	identical to	
\sim	approximately equal	weak approximation	$996 \sim 1000$
\approx	approximately equal	approximation	$\sin(0.01) \approx 0.01$
\propto	proportional to	proportional to	$F \propto a$

∞	lemniscate	infinity symbol	
\ll	much less than	much less than	$1 \ll 1000000$
\gg	much greater than	much greater than	$1000000 \gg 1$
$()$	parentheses	calculate expression inside first	$2 * (3+5) = 16$
$[]$	brackets	calculate expression inside first	$[(1+2)*(1+5)] = 18$
$\{ \}$	braces	set	
$\lfloor x \rfloor$	floor brackets	rounds number to lower integer	$\lfloor 4.3 \rfloor = 4$
$\lceil x \rceil$	ceiling brackets	rounds number to upper integer	$\lceil 4.3 \rceil = 5$
$x!$	exclamation mark	factorial	$4! = 1*2*3*4 = 24$
$ x $	single vertical bar	absolute value	$ -5 = 5$
$f(x)$	function of x	maps values of x to f(x)	$f(x) = 3x+5$
(a,b)	open interval	$(a,b) = \{x \mid a < x < b\}$	$x \in (2,6)$
$[a,b]$	closed interval	$[a,b] = \{x \mid a \leq x \leq b\}$	$x \in [2,6]$
Δ	delta	change / difference	$\Delta t = t_1 - t_0$
Δ	discriminant	$\Delta = b^2 - 4ac$	
Σ	sigma	summation - sum of all values in range of series	$\sum x_i = x_1 + x_2 + \dots + x_n$
Π	capital pi	product - product of all values in range of series	$\prod x_i = x_1 \cdot x_2 \cdot \dots \cdot x_n$
e	e constant / Euler's number	$e = 2.718281828\dots$	$e = \lim (1+1/x)^x, x \rightarrow \infty$
ϕ	golden ratio	golden ratio constant	$\phi = 1.61803398875$

Logic symbols

Symbol	Symbol Name	Meaning / definition	Example
\cdot	and	and	$x \cdot y$
\wedge	caret / circumflex	and	$x \wedge y$
$\&$	ampersand	and	$x \& y$
$+$	plus	or	$x + y$
\vee	reversed caret	or	$x \vee y$

	vertical line	or	$x y$
!	exclamation mark	not - negation	$! x$
\oplus	circled plus / oplus	exclusive or - xor	$x \oplus y$
\sim	tilde	negation	$\sim x$
\Rightarrow	implies		
\Leftrightarrow	equivalent	if and only if	
\forall	for all		
\exists	there exists		
\nexists	there does not exist		
\therefore	therefore		
\because	because / since		

Calculus & Analysis symbols

Symbol	Symbol Name	Meaning / definition	Example
$\lim_{x \rightarrow x_0} f(x)$	limit	limit value of a function	
ε	epsilon	represents a very small number, near zero	$\varepsilon \rightarrow 0$
e	e constant / Euler's number	$e = 2.718281828\dots$	
$\frac{dy}{dx}$	derivative	derivative - Lagrange's notation	$d(3x^3)/dx = 9x^2$
\int	integral	opposite to derivation	
$[a,b]$	closed interval	$[a,b] = \{x \mid a \leq x \leq b\}$	
(a,b)	open interval	$(a,b) = \{x \mid a < x < b\}$	
i	imaginary unit	$i \equiv \sqrt{-1}$	$z = 3 + 2i$
z^*	complex conjugate	$z = a+bi \rightarrow z^*=a-bi$	$z^* = 3 - 2i$
∇	nabla / del	gradient / divergence operator	$\nabla f(x,y,z)$
\vec{x}	vector		
\hat{x}	unit vector		

CHAPTER 1 – SETS

Answer the following:

1. Let $A = \{1, 0, \{1, \emptyset\}, \emptyset, \{\emptyset\}, 2\}$. State whether the following statements are true or false. Also justify your answer.

(i) $1, 0 \in A$	(ii) $\{1, \emptyset\} \subset A$	(iii) $\{1, \emptyset\} \in A$	(iv) $\{\emptyset\} \subset A$
(v) $\{\emptyset\} \in A$	(vi) $\{\{\emptyset\}\} \subset A$	(vii) $\{1, \emptyset, 0\} \subset A$	(viii) $\emptyset \subset A$
(ix) $\{2, \emptyset\} \in A$	(x) $\{\{\emptyset\}, \{1\}\} \subset A$		

2. Write the complement of the following sets by considering \mathbf{R} as the universal set:

(i) $(2, 4]$	(ii) $\{x: x \in \mathbf{R}, 2x+1 > 7\}$
(iii) $\{0\}$	(iv) $\{x: x \in \mathbf{R}, 0 < x < 3\}$

3. Write all proper subsets, if possible, of the following sets

a) $\{1, \{1\}\}$	b) $\{\emptyset\}$	c) $\{x, y, z\}$	d) $\{1, 2, 3, 4\}$
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4. With the help of Venn diagram prove that

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

5. For any two sets prove that $A \cup B = A \cap B \Leftrightarrow A = B$

6. Let $U = \{1, 2, 3, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 7\}$, $B = \{2, 5, 6, 7\}$ and $C = \{1, 7, 8, 9\}$. Verify the following

(i) $A - (B \cup C) = (A - B) \cap (A - C)$
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

7. Verify De Morgan's laws for $U = \{2, 3, 4, 5, 6, 7, 9\}$, $A = \{3, 7\}$ and $B = \{2, 5, 7\}$

8. In survey 600 television viewers it was found that 275 watch cricket, 200 watch football, 150 watch hockey, 50 watch cricket & football, 30 watch football & hockey, 30 watch cricket & hockey and 75 do not watch any of the three games. How many watch

(i) All the three games
(ii) Exactly one game
(iii) Neither cricket nor football
(iv) Cricket and football but not hockey
(v) Cricket or Football but not Hockey
(vi) Either Cricket or Hockey
(vii) Exactly two games

- (viii) At most one game
 (ix) At least two games ?

9. A market research group conducted a survey of 2000 consumers and reported that 1650 consumers like product A and 1350 consumers like product B. What is the least and maximum number that must have liked both of the products? [ans : 1000, 1350]

10. Draw the Venn diagram for (i) $(A \cup B) - C$ (ii) $(A \cap B) \cup C$

11. Using Venn diagram show that $A \cap B' = A - B$

12. Find $n(P(P(P(\emptyset))))$ ans : 4

13. Out of 30 students ; 15 passed in English, 12 in Mathematics , 8 in science, 6 in English and mathematics , 7 in mathematics and science, 4 in English and science, 4 in all three. Find, how many passed in

- (i) English and mathematics but not in science
 (ii) Mathematics and science but not in English
 (iii) Mathematics only
 (iv) More than one subject
 (v) None of the three subjects. [Ans: 2, 3, 3, 9, 8]

CHAPTER 2 – RELATIONS AND FUNCTIONS

Answer the following:

- Let $A = \{1,2,3\}$, $B = \{4,5,6\}$, $C = \emptyset$ and $D = \{3,4\}$
 Find (i) $A \times (B \cup C)$ (ii) $D \times (B \cap C)$
 (iii) $(A - C) \times (B \cap C)$ (iv) $(B \times D) \cap (D \times B)$
- $A = \{1,0,-1\}$, $B = \{2,3\}$, $C = \{1,-1\}$, find $A \times B \times C$ and $B \times A \times C$
- If $(a,1)$, $(b,-1)$ and (c,c) are three elements of $A \times B$ and $n(B \times A) = 9$, Find the sets A, B and $B \times A$.
- Write all relations from A to B where $A = \{3, 1\}$ and $B = \{2,5\}$.

5. If $A = \{1, 2\}$, find the power set of $A \times A$
6. How many possible relations are defined on $A = \{0, 1\}$. Write all relations in the roster form
7. Let $R = \{(x + 1, x^2) : x \in \{0, 1, 2, 3, 4, 5\}\}$.
- (i) Write R in Roster form (ii) Draw the arrow diagram of R
- (iii) Find the domain of R (iv) Find the range of R.
8. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by the rule $(a, b) \in R$ if $1 + ab > 0$. Write R explicitly.
9. What is the fundamental difference between a relation and a function? Is every relation a function? Justify your answer. Is the converse true?
10. What is the domain and range of the following relations
- (i) $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } x + y = 10\}$
- (ii) $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } x^2 + y^2 \leq 20\}$
11. Draw the graph of the following functions. Also, find the range and domain in each case
- (i) $f(x) = \begin{cases} x + 1, & x > 1 \\ 1, & x = 1 \\ x - 1, & x < 1 \end{cases}$
- (iii) $f(x) = \begin{cases} |x|, & x \geq -2 \\ 4, & x < -2 \end{cases}$
- (iv) $f(x) = \{x\}$, where $\{x\} = x - [x]$, is called fractional part function of x.
- (v) $f(x) = \begin{cases} 3 - x, & x > 1 \\ 1, & x = 1 \\ 2x, & x < 1 \end{cases}$
- (vi) $f(x) = \begin{cases} x, & x \geq 2 \\ x^2, & x < 2 \end{cases}$

12. Find the domain and range of the following functions:

$$(i) f(x) = \sqrt{25 - x^2}$$

$$(ii) f(x) = \sqrt{x^2 - 25}$$

$$(iii) f(x) = \sqrt{x - 7}$$

$$(iv) f(x) = \sqrt{7 - x}$$

$$(v) f(x) = |x - 1|$$

$$(vi) f(x) = -|x + 3|$$

$$(vii) f(x) = 4 - |x + 2|$$

$$(viii) f(x) = \frac{1}{\sqrt{x-6}}$$

$$(ix) f(x) = \frac{x^2+1}{x^2-2}$$

$$(x) f(x) = \frac{1}{x^2-3x+2}$$

13. Let g and f be two functions defined by $f(x) = \sqrt{x - 7}$ and $g(x) = \sqrt{x^2 - 1}$, describe (i) $f + g$ (ii) fg (iii) $g - f$ (iv) $\frac{f}{g}$ (v) $\frac{g}{f}$

CHAPTER 3 – TRIGONOMETRIC FUNCTIONS

Answer the following:

1. Find the value of $\frac{\cos(2\pi + \theta) \operatorname{cosec}(2\pi - \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\sec\left(\frac{9\pi}{2} + \theta\right) \cos(4\pi - \theta) \cot(3\pi - \theta)}$

2. Evaluate $\cos 15^\circ - \sin 15^\circ$

3. If $\cos \alpha = \frac{13}{14}$ and $\cos \beta = \frac{1}{7}$ where α and β are acute angles, show that $\alpha - \beta = \frac{\pi}{3}$

4. The circular measures of two angles of a triangle are $\frac{1}{2}$ and $\frac{1}{3}$, find the third angle in English system [ans : $132^\circ 16' 22''$]

5. A circular wire of radius 3 cm is cut and bent so as to lie along a circle of radius 48cm. Find the angle subtended by the wire at the centre of the circle.

[ans : 22.5°]

6. Find the angle between the hands of a clock at 7:20 P.M. [ans: 100°]

7. If $\tan x = \frac{m}{m-1}$ and $\tan y = \frac{1}{2m-1}$, prove that $x - y = \frac{\pi}{4}$

8. If $\sec x + \tan x = 4$, find $\sin x$ and $\cos x$. Also find the quadrant in which x lies.

[ans : $\sin x = 15/17$, $\cos x = 8/17$, First]

9. If $\sin \alpha + \sin \beta = a$, $\cos \alpha + \cos \beta = b$, prove that $\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$

$$\text{and } \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

10. If $\tan A + \cot A = 2$, then find $\tan^{100} A + \cot^{100} A$ ans : 2

11. Prove that $\frac{\cos 2x \cos 3x - \cos 2x \cos 7x + \cos x \cos 10x}{\sin 4x \sin 3x - \sin 2x \sin 5x + \sin 4x \sin 7x} = \cot 6x \cot 5x$

12. Prove that $\tan \frac{\pi}{20} \tan \frac{3\pi}{20} \tan \frac{5\pi}{20} \tan \frac{7\pi}{20} \tan \frac{9\pi}{20} = 1$

13. Prove that $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = -\frac{1}{2}$

14. Prove that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

15. Prove that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

16. Prove that $\frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} + \frac{\sin(A-B)}{\cos A \cos B} = 0$

17. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{1}{4}$ prove that $\tan\left(\frac{x+y}{2}\right) = \frac{1}{2}$

18. Prove that $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8x}}} = 2\cos x$

19. Prove that $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

20. Prove that $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

21. Solve the equation $\cos x + \sin x = \cos 2x + \sin 2x$

22. Find all values of A between 0° and 720° which satisfy the equation

$$2\cos^2 A - 5\cos A + 2 = 0 \quad [\text{ans : } 60^\circ, 300^\circ, 420^\circ, 660^\circ]$$

23. Find the value of $\sin 2x$, $\sin \frac{x}{2}$, $\cos 2x$, $\cos \frac{x}{2}$

if, $\cos x = \frac{-3}{5}$, x lies in third quadrant.

24. Find the principal and general solution of

(a) $2\sin x + 1 = 0$

(b) $\sin x + \cos x = 0$

25. Find the general solutions for the following

a) $\sin 2x + \sin 4x + \sin 6x = 0$

b) $2\cos^2 x + 3\sin x = 0$

c) $\cos x + \cos 3x = 2 - 4\sin^2 x$

d) $\cos x - \sin x = \frac{1}{\sqrt{2}}$

e) $\sqrt{2} \sec x + \tan x = 1$

CHAPTER 4 : PRINCIPLE OF MATHEMATICAL INDUCTION

Answer the following:

1. Let $P(n) : "n^2 - n + 41 \text{ is prime}"$. Show that $P(1)$ and $P(2)$ are true, but $P(41)$ is not true.
2. Let $P(n) : "10n + 3 \text{ is prime if } n \text{ is not divisible by } 3"$. Give an example to show that $P(n)$ is not true for all $n \in \mathbb{N}$.

Using the Principle of Mathematical Induction prove that for all $n \in \mathbb{N}$ (Q.3- 16):

$$3. 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$4. 3 \cdot 6 + 6 \cdot 9 + 9 \cdot 12 + \dots + 3n(3n+3) = 3n(n+1)(n+2)$$

$$5. a + (a+d) + (a+2d) + \dots + (a+(n-1)d) = \frac{n}{2} [2a + (n-1)d]$$

$$6. 4 + 8 + 12 + \dots \text{to } n \text{ terms} = 2n(n+1)$$

$$7. n(n+1)(n+2) \text{ is divisible by } 6$$

$$8. n(n+1)(2n+1) \text{ is a multiple of } 6$$

$$9. n^2 + n \text{ is even}$$

$$10. (n+3)^2 \leq 2^{n+3}$$

$$11. 2 \cdot 7^n + 3 \cdot 5^n - 5 \text{ is divisible by } 24$$

$$12. 11^{n+2} + 12^{2n+1} \text{ is divisible by } 133$$

$$13. n^3 + (n+1)^3 + (n+2)^3 \text{ is divisible by } 9$$

$$14. \text{Whenever } 3^{2n} \text{ is divided by } 8 \text{ the remainder is always } 1.$$

$$15. \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15} \text{ is a natural number}$$

CHAPTER 5: COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Answer the following:

$$1. \text{ Write the number } \frac{3+\sqrt{-1}}{2-\sqrt{-1}} \text{ in the standard form. Also find its modulus.}$$

$$2. \text{ Simplify } 1 + i^{10} + i^{20} + i^{30}$$

$$3. \text{ Express } 3i^3 + 6i^{16} - 7i^{29} + 4i^{27} \text{ in the form } x + iy$$

$$4. \text{ If } (3-4i)(x+iy) = 1, \text{ find } x \text{ and } y.$$

5. If $1 + 4\sqrt{3}i = (a + ib)^2$, prove that $a^2 - b^2 = 1$ and $a = 2\sqrt{3}b$.

6. Find the real part of $\frac{1}{1 - \cos x + i \sin x}$.

7. Express the following into the form $a + ib$:

$$(i) \quad \frac{5+2i}{-1+i\sqrt{3}} \quad (ii) \quad \frac{2+i}{(1+i)(1-2i)} \quad (iii) \quad \frac{(1-i)(2-i)(3-i)}{1+i}$$

8. Find the complex conjugate of the following:

$$(i) \quad (2 + 5i) \quad (ii) \quad \left(\frac{1-i}{1+i}\right)^{100}$$

9. If $x + iy = \frac{a+ib}{a-ib}$, show that $x^2 + y^2 = 1$.

10. If $3 + ix^2y$ and $x^2 + y + 4i$ are complex conjugates of each other, find x and y .
[ans : $x = -2$ or 2 , $y = -1$]

11. Show that $\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}} + \frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}$ is purely real

12. If $a = \cos x + i \sin x$, then show that $\frac{1+a}{1-a} = i \cot\left(\frac{x}{2}\right)$

13. Find x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+1}{3-i} = i$ (ans: $3, -1$)

14. Solve the following quadratic equations:

$$(i) \quad ix^2 - x + 12i = 0 \quad (\text{ans: } -4i, 3i)$$

$$(ii) \quad x^2 - (3\sqrt{2} - 2i)x - 6\sqrt{2}i = 0 \quad (\text{ans: } -2i, 3\sqrt{2})$$

$$(iii) \quad 2x^2 + 3ix + 2 = 0 \quad (\text{ans: } -2i, i/2)$$

CHAPTER 6: LINEAR INEQUALITIES

1. Solve the following

$$(i) \quad \frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}$$

$$(ii) \quad \frac{1}{2} \left(\frac{3x}{5} + 4 \right) > \frac{1}{3} (x - 6)$$

$$(iii) \quad \frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$$

$$(iv) \quad \frac{2x-1}{12} - \frac{x-11}{3} \leq \frac{3x+1}{4}$$

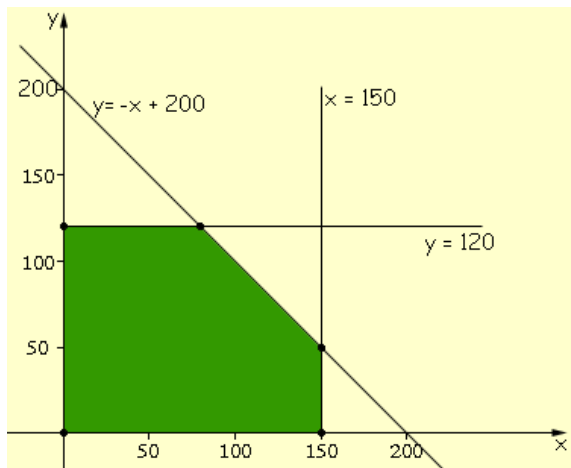
2. Solve the following system of inequalities and show the solutions on the number line.

$$(i) \quad x + 2 \leq 5 ; 3x - 4 > -2 + x$$

$$(ii) \quad 2(2x + 3) - 10 < 6(x - 2) ; \frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$$

3. Solve $|x| > 8$ and represent the solution set on the number line.

4. Determine the system of linear inequations for which the solution set is the shaded region in the following diagram.



5. Solve the following system of inequalities graphically:

$$x + y \leq 5 ; \quad 4x + y \geq 4 ; \quad x + 5y \geq 5 ; \quad x \leq 4 ; \quad y \leq 3$$

6. A furniture dealer deals in only two items: tables and chairs. He has Rs. 15000 to invest and a space to store at most 60 pieces. A table costs him Rs.750 and chair Rs. 150. Formulate the data in the form of inequalities and draw a graph representing the solution of the inequalities.

CHAPTER 7 PERMUTATIONS AND COMBINATIONS
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Answer the following:

1. Show that $32!$ is divisible by 2^{31}
2. How many numbers are there between 100 and 1000 in which all the digits are distinct? (ans: 648)
3. Prove that ${}^n P_r = {}^{n-1} P_r + r \cdot {}^{n-1} P_{r-1}$
4. In how many ways can 4 books on Mathematics and 3 books on English be placed on a shelf so that books on the same subject always remain together (ans: 288)
5. The letters of the word **ORIENTAL** are arranged in such a manner that vowels and consonants occur alternately. In how many different ways can it be done? (ans: 1152)
6. How many different numbers, each of six digits can be formed by using the digits 1 , 2 , 1 , 2 , 0 , 2 ? (ans : 50)
7. Find n , if ${}^{2n} C_1$, ${}^{2n} C_2$ and ${}^{2n} C_3$ are in A.P ? (ans : $n = 7/2$)

8. A committee of 6 is to be formed from 6 boys and 4 girls. In how many ways can this be done if the committee contains (i) 2 girls (ii) at least 2 girls?
9. A polygon has 44 diagonals, find the number of sides. (ans : 11)
10. Find the number of rectangles in the following

(ans : 150)

11. If ${}^n C_r : {}^n C_{r+1} = 1 : 2$ and ${}^n C_{r+1} : {}^n C_{r+2} = 2 : 3$ find n and r . (ans: 14, 4).
12. Twelve persons meet in a room and each shakes hand with all the others. Find the numbers of handshakes. (ans : 66)
13. In an examination, a candidate has to pass in each of the 4 subjects. In how many ways can he fail? (ans: 15)
14. How many different words can be formed by using all the letters of the word ALLAHABAD.
- (i) In how many of them, vowels occupy the even positions?
- (ii) In how many of them both L do not come together?

(ans : 60 , 5880)

Value Based Questions

Q.1 Consider the following sets:

$P = \{ x: x \text{ is a letter in the word 'DISCIPLINE'} \}$

$Q = \{ x: x \text{ is a letter in the word 'INDISCIPLINE'} \}$

$R = \{ x: x \text{ is a letter in the word 'OBEDIANCE'} \}$

$S = \{ x: x \text{ is a letter in the word 'HONESTY'} \}$ and

$T = \{ x: x \text{ is a letter in the word 'PATRIOTISM'} \}$

- (i) *From the above sets, choose the pairs of equal sets*
- (ii) *From the above sets, choose the pairs of equivalent sets*
- (iii) *Is the collection of all undisciplined students in a class, a set?*
- (iv) *Is the set R, a subset of P?*

Q.2 Let $A = \{ \text{Raj, Shruti, Manoj, Rachna} \}$ and $B = \{ \text{honesty, discipline, arrogance} \}$. Consider the subset $R = \{ (\text{Raj, honesty}), (\text{Shruti, discipline}), (\text{Manoj, arrogance}), (\text{Ram, discipline}), (\text{Rachna, honesty}) \}$ of $A \times B$ and answer the following:

- (i) *Is Ram honest and disciplined?*
- (ii) *Is Rachna arrogant?*
- (iii) *Is Manoj disciplined?*
- (iv) *Is R, a function from A to B?*

Q3. Two students A and B of class XI answered the following question in the paper of mathematics

‘Find the maximum value of $\sin^2 x + \cos^4 x$ ’ as follows

‘A’ found the maximum value as 2, for, maximum value of $\sin^2 x$ is 1 and also that of $\cos^4 x$ is 1.

‘B’ found the maximum value as 1, for,

$$\sin^2 x + \cos^4 x = \sin^2 x + \cos^2 x (1 - \sin^2 x) = 1 - \sin^2 x \cos^2 x.$$

The teacher awarded full marks to B and zero to A.

- (i) *Is the teacher justified in awarding full marks to B and zero to A. Solve the question and give reasons.*
- (ii) *How will you respond as a student?*

Q.4. On a particular day , class XI teacher of a school sent the following report to the principal:

(a) roll no. 1 is absent

(b) if roll number n is absent then roll number $n + 1$ is also absent for $n < 40$

(i)if there are 50 students in the class , find the number of students present in the class on that particular day.

(ii)On the basis of which principle, you have drawn the above conclusion.

(iii)Comment on the regularity of the students of the class.

Q.5. Raghu tells Dev that his age (in years) is equal to $|5 - 12i|$. Further, he asserts that after three years, his age will be equal to $|8 + 15i|$.

(i) Find the age of Raghu as per his first statement.

(ii) Find the age of Raghu after 3 years as per the second statement.

(iii) Comment on the behaviour of Raghu.

Q.6. Eash and Eashan are twins . Eash says that his age (in years) when subtracted from its square lies between 182 and 240. Eashan says that his age when added to its square lies between 210 and 272.

(i) Do the statements made by Eash and Eashan agree with each other

(ii) Write the system of linear inequations giving an idea about their age.

(iii) Comment on the behaviour of Eash and Eashan.

Q.7. A school wants to award its students for the values of Honesty and hard work with a total cash of ₹ 6,000. The award for Honesty should be atleast ₹ 2,000 and for hard work it should be at most ₹ 3,500. Solve the above problem graphically. Apart from these values, suggest two more values which the school must include for awards.

Q8. License plates of cars in South Delhi bear the code DL 3C followed by a sequence of 2 letters and 4 digits (except 0000). Find the number of possible license plates that can be formed with such codes. According to you why number plates are mandatory in vehicles?

[Ans: 67,59, 324]

HOTS Questions

1. Root(s) of the equation $2\sin^2 x + \sin^2 2x$ are? **IIT - 2009**

2. Find the value of b for which the equations

$$x^2 + bx - 1 = 0$$

$x^2 + x + b = 0$ have a common solution (ans: $-i\sqrt{3}$) **IIT - 2011**

3. How many natural numbers less than a million can be forms using the natural numbers 0, 7 and 8?

Choose the right answer : [724, 726, 728, 730] **SCRA 2012**

4. For $x^2 + x - r = 0$ the roots are, a and b such that $a^3 + b^3 = 6$, then the

value of r is (ans: $\frac{-5}{3}$) **CPT -2011**

5. What is the number of ways of arranging the letters of the word 'BANANA'

so that no two N's appear together? (ans : 40) **NDA 2010**

6. If 100 times the 100th term of an AP with non zero common difference

equals the 50 times its 50th term, then the 150th term of this AP is:-----

(ans: 0) **AIEEE 2012**

7. Find the range of the function $f(x) = \frac{|x-1|}{x-1}$. **CBSE 2010**

8. Determine the numbers a , b , c such that the function $y = ax^2 + bx + c$ fits to

the data points $(-1, 7)$, $(0, 4)$ and $(2, 6)$. Hence find the function.

$$\text{Ans: } \frac{4}{3}x^2 - \frac{5}{3}x + 4$$

CA Foundation 2006

9. What comes next in the following sequence ?

1, 4, 5, 6, 7, 9, 11 ,

ans: 100 (why)

10. Legs Problem

'7 girls on a bus being driven to school

each girl has 7 bags

for each bag there are 7 big cats

for every big cat there are 7 little cats'

How many legs are on the bus?

Ans :10992.

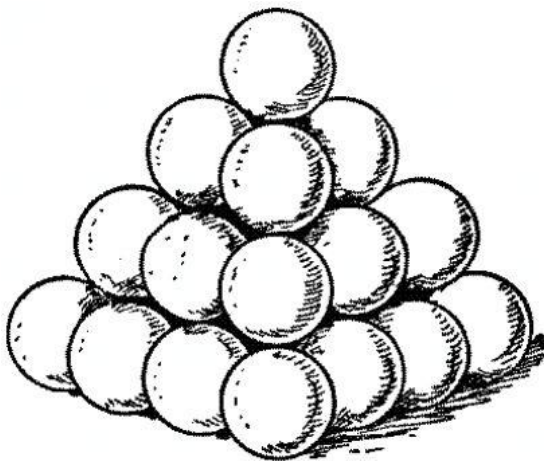
11. Civil Service puzzles

(i) If you had three apples and four oranges in one hand and four apples and three oranges in the other hand, what would you have?

(ii) How can you drop a raw egg from a height of 15cm onto a concrete floor without cracking it?

(iii) If it took eight men ten hours to build a wall, how long would it take four men to build it?

12. The Artillery men's Dilemma Puzzle



How many cannon balls are there in the square pyramid?

Greek alphabet letters

Greek Symbol		Greek Letter Name	English Equivalent	Pronunciation
Upper Case	Lower Case			
A	α	Alpha	a	al-fa
B	β	Beta	b	be-ta
Γ	γ	Gamma	g	ga-ma
Δ	δ	Delta	d	del-ta
E	ε	Epsilon	e	ep-si-lon
Z	ζ	Zeta	z	ze-ta
H	η	Eta	h	eh-ta
Θ	θ	Theta	th	te-ta
I	ι	Iota	i	io-ta
K	κ	Kappa	k	ka-pa
Λ	λ	Lambda	l	lam-da
M	μ	Mu	m	m-yoo
N	ν	Nu	n	noo
Ξ	ξ	Xi	x	x-ee
O	ο	Omicron	o	o-mee-c-ron
Π	π	Pi	p	pa-yee
P	ρ	Rho	r	row
Σ	σ	Sigma	s	sig-ma
T	τ	Tau	t	ta-oo
Υ	υ	Upsilon	u	oo-psi-lon
Φ	φ	Phi	ph	f-ee
X	χ	Chi	ch	kh-ee
Ψ	ψ	Psi	ps	p-see
Ω	ω	Omega	o	o-me-ga

Roman numerals

Number	Roman numeral
1	I
2	II
3	III
4	IV
5	V
6	VI
7	VII
8	VIII
9	IX
10	X
11	XI
12	XII
14	XIV
15	XV
16	XVI
19	XIX
20	XX
30	XXX
40	XL
50	L
60	LX
70	LXX
80	LXXX
90	XC
100	C
200	CC
300	CCC
400	CD
500	D
600	DC
700	DCC
800	DCCC
900	CM
1000	M