DON BOSCO SCHOOL ALAKNANDA

ASSIGNMENT: 1 **CLASS XII** **MATHEMATICS**

CHAPTER: RELATIONS AND FUNCTIONS

POINTS TO REMEMBER:

☺ A Relation R from a set A to a set B is a subset of A × B.
If n(A) = r, n (B) = s from set A to set B then n (A × B) = rs. and no. of relations from A to B = 2^{rs}.

☺ Empty relation: R = ϕ is a relation defined on set A, ϕ⊂A×A, also known as the void (null) relation.
Universal relation: R = A×A is known as universal relation.
Identity relation: R = {(x,y): x,y∈A, x=y}
Inverse relation: let R: A→B, then R^{-1}:B→A defined by {(y,x): (x,y)∈R}

☺ **Reflexive Relation:** Relation R defined on set A is said to be reflexive iff (a,a) ∈R (a R a) ∀ a∈A.
Symmetric Relation: Relation R defined on set A is said to be symmetric
iff (a, b) ∈ R ⇒ (b, a) ∈ R (a R b ⇒ b R a) ∀ a, b ∈ A
Transitive Relation : Relation R defined on set A is said to be transitive
if (a, b) , (b, c) ∈ R ⇒ (a, c) ∈ R (a R b and b R c ⇒ a R c) ∀ a, b, c ∈ R
Equivalence Relation : A relation defined on set A is said to be an equivalence relation iff it is reflexive, symmetric and transitive.

☺ **One-One Function (injective):** f: A→B is said to be one-one if distinct elements in A has distinct images in B.
i.e. if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in A$ OR if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \ \forall \ x_1, x_2 \in A$
Onto function (surjective): A function f: A →B is said to be onto iff R_f = B (Co Domain). (Atleast one pre-image)
i.e. For every y∈B, there exist x∈A s.t. f(x) = y
Bijective Function: A function which is both injective(1-1) and surjective(onto) is called bijective.

☺ A and B are finite sets. F: A→B is a function.
f is one-one ⇒ n(A)≤ n(B); f is onto ⇒ n(A)≥ n(B); f is bijective⇒ n(A)= n(B);

☺ If n(A) = p and n(B) = q; no. of functions from A to B = q^p.

☺ Number of 1-1 and onto functions from a finite set A to finite set B = m!, where n(A) = n(B) = m. otherwise zero.

☺ **Composition of Two Functions:** If f : A → B, g : B → C are two functions, then composition of f and g denoted by gof is a function from A to C given by, (gof) (x) = g(f (x)) ∀ x ∈ A
Clearly gof is defined if Range of f ⊆ domain of g. ($R_f \subseteq D_g$) Similarly fog can be defined.

☺ **Invertible Functions:** A function f : X → Y is invertible iff it is bijective.
If f: X→Y is bijective function, then function g:Y →X is said to be inverse of f and is denoted by f^{-1}.
iff fog = I_y and gof = I_x where I_x, I_y are identity functions on X and Y resp.

☺ Number of binary operations on A with n(A) = m is equal to m^{m^2}

☺ **Binary Operation:** A binary operation ∗defined on set A is a function from A × A → A. ∗(a, b) = a ∗ b.
i.e. a∗b ∈A ∀ a,b ∈A. (closure property)
Binary operation ∗ defined on set A is said to be commutative iff a ∗ b = b ∗ a ∀ a, b ∈ A.
Binary operation ∗ defined on set A is called associative iff a ∗(b ∗ c) = (a ∗ b) ∗ c ∀ a, b, c ∈ A
For a binary ∗ on A, a unique element e ∈A is said to be the identity element iff a ∗ e = e ∗ a ∀ a∈A.
If ∗ is binary operation on A, then an element b is said to be inverse of a ∈ A iff a ∗ b = e = b ∗ a
Inverse of an element, if it exists, is unique.

ASSIGNMENT QUESTIONS

1. Show that the relation R in the set A ={x∈W, 0≤x≤17} given by R={(a,b): |a−b| is a multiple of 5} is an equivalence relation. Also find [3] and [2].
2. Is the relation R in the set A={1, 2, 3, 4, 5} defined as R ={(a, b) : b = a + 1} reflexive or transitive ?
3. Let R be a relation on R defined by {(a,b):a+b is even}.Is R an equivalence relation?

4. Prove that f: R→R given by $f(x) = x^3+3$ is a bijection. Find $f^{-1}(x)$. Also find $f^{-1}(-5)$ and $f^{-1}(30)$.
Does truthfulness and honesty have any relation?
5. Is the relation R on R of real numbers defined by R ={(a,b): 1+ab>0, a,b ϵR }, reflexive, symmetric and transitive. Justify your answer.
6. Let R ={(1,1),(2,2),(2,1),(3,3),(3,2)} be a relation on A={1,2,3}. Determine whether R is reflexive, symmetric and transitive. Justify your answer.
7. Let R be a relation from A = {2, 3, 4, 5} to B = {3, 6, 7, 10} given by x R y iff x is relatively prime to y. find the domain and range of R. Determine whether R is reflexive, symmetric and transitive. Justify your answer.
8. Show that the relation R on the set N×N defined by (a,b)R(c,d) ⇔ ad=bc, is an equivalence relation.
9. If n(A) = n(B) = 3, How many bijective functions can be formed from A to B?
10. If f : A → B is bijective function such that n (A) = 10, then n (B) = ?
11. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, show that $f\left(\frac{2x}{1+x^2}\right) = 2\,f(x)$
12. Show that f:Z→Z defined by $f(x)=x^2+x$ is many-one and into (neither 1-1 nor onto)
but f: R→R defined by $f(x)=x^3+x$ is a bijection.
13. Let f:Z→Z defined by $f(x)=\begin{cases} 2x+1, & \text{if x is odd} \\ 2x-1, & \text{if x is even} \end{cases}$. Find (a) f(3) (b) f(4) (c) x when f(x) =−9.
Is this function one-one and onto.
14. Let f, g: R→R be two functions. Consider the product function f×g: R→R. With the help of an example show that
(a) if both f and g are onto, f×g need not be onto.
(b) if both f and g are one-one, f×g need not be one-one.
15. Determine whether f: R→R defined by f(x) = |2x−3|is 1-1 and onto.
16. Is the function f: N→N defined by $f(m)=m^2+m+2$ one-one?
17. If f (x) = x + 1, g(x) = x – 1, Then find the value of (gof)(3)
18. If f,g: R→R defined by f(x)= 3x+2 and $g(x)=\frac{x}{x^2+1}$ for all xϵR, find the following:
(a) fof (b) fog (c) gof (d) gog
19. f: R→R defined by f(x)= ax +b ∀ xϵR, then find the constants a,b such that fof=I_R.
20. Let f: N→Y defined as $f(x)=4x^2 + 12x +15$, where Y is range of f(x). Show that f is invertible. Also find $f^{-1}(x)$.
21. If f: R→R defined by f(x)= 5x −3, find g: R→R such that fog= I_R =gof
22. If f={(1,5),(2,3),(3,0),(4,−2)}, find f^{-1} .
23. Prove that f: R→R defined by $f(x) = \frac{2x-7}{4}$ is invertible. Find $f^{-1}(x)$.
24. Let * be a binary operation on R defined by a* b = $\sqrt{a^2+b^2}$ ∀a,b ϵ R. Examine whether * is binary or not. Discuss the associativity and commmutativity of *. Also find the identity element and invertible elements, if any in R.
25. Let * be a binary operation on R defined by a* b =$\frac{3ab}{7}$. Examine whether * is binary or not. Discuss the associativity and commmutativity of *. Also find the identity element and invertible elements, if any in R.
26. Let A={1,2,3,4,5} and * be a binary operation on A defined by a* b=max(a,b). construct the operation/composition table for *.Prove that * is commutative and associative. Find the identity element. Also find all invertible elements.
27. Determine whether the relation R={(x,y): x and y are honest.} in a set A of students in a school at a particular time is symmetric and transitive. What are the advantages of students being honest?
28. Let A={a_1, a_2, a_3, a_4, a_5} and B={ b_1, b2, b_3, b_4} where a_i, b_i represent school going children. Define a relation from A to B by x R y iff y is a true friend of x. If R = {(a_1, b_1),(a_2, b_1), (a_3, b_3), (a_4, b_4), (a_5, b_2)}; determine whether R is bijective or not. Justify your answer. Do you think true friendship is important in life? Give two reasons.
29. Prove that a function f : N→Z defined by $f(n) = \begin{cases} \frac{n-1}{2} & n \text{ is odd} \\ -\frac{n}{2} & n \text{ is even} \end{cases}$ is bijective. Find its inverse.
30. Let ∗ be an operation on Q – { – 1}defined by a∗ b = a + b + ab. Check whether ∗ is binary, commutative or associative. Find the identity element, if any. Also find the invertible elements and their inverses.

31. Let g(x) = 1+ x – [x] and f(x) = $\begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$ find fog and gof.
32. If f: R →(0,2) defined by f(x) = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ + 1 is invertible, find $f^{-1}(x)$.
33. Find $f^{-1}(x)$, if f: [1,∞)→[1,∞) defined by f(x) = $2^{x(x-1)}$ is invertible.
34. Let R = { (a,b): $a^2 + b^2 = 1$} be a relation on **R**. Check whether R is reflexive, symmetric or transtive.
35. Prove that the relation " congruence modulo m" on Z is an equivalence relation.
36. If R and S are two equivalence relations on a set A, prove that R ∩ S is also an equivalence relation.
37. Show that union of two equivalence relations is not necessarily an equivalence relation.
38. Prove that R^{-1} is an equivalence relation if R is an equivalence relation.
39. R is a relation on Z given by R= { (a,b): |a – b|≤ 1}. Determine whether R is reflexive, symmetric and transitive.
40. Let A = Q×Q and let ∗ be a binary operation on A defined by (a,b) ∗(c,d) = (ac, b+ad). Determine whether ∗ is commutative and associative. Find the identity element in A. Find the invertible elements of A.
41. Let A = N×N and let ∗ be a binary operation on A defined by (a,b) ∗(c,d) = (ad+bc,bd). Determine whether ∗ is commutative and associative. Show that A has no identity element.
42. Consider a set S= {1, 2, 3, 4, 5, 6}. Define a binary operation ∗ on S as a∗b = r, where r is the least non-negative remainder when ab is divided by 7. Determine whether ∗ is binary, commutative and associative. Find the identity element and invertible elements. Also find the inverse.
43. Let X be a non-empty set and let ∗ be a binary operation on P(X) defined by A∗B = A∪B. Prove that ∗ is commutative and associative. Find the identity element and invertible elements.
44. Show that f: R⟶(−1 , 1) given by f(x) = $\frac{-x|x|}{1+x^2}$ is invertible. Also find $f^{-1}(x)$.
45. Is f: [1,∞)⟶[2,∞) given by f(x) = x + $\frac{1}{x}$ bijective? If yes, find $f^{-1}(x)$.
46. Find the value of a for which f(x) – 1 + ax is the inverse of itself.
47. Let f(x) = $\begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$ find fof.
48. Let f(x) = $\frac{3x-2}{2x-3}$, find fof in R−$\left\{\frac{3}{2}\right\}$. Hence find its inverse.
49. Let f : $R^+ \to R^+$ given by f(x) = e^x. is f invertible? If yes, find $f^{-1}(x)$.
50. Let f: A⟶B, g: B⟶A be two functions such that gof = I_A. Prove that f is one-one and g is onto.
51. Let f: A⟶B, g: B⟶C such that gof is 1-1 and f is onto, prove that g is 1-1.
52. If f: R⟶ R given by f(x) = $\sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \ \cos\left(x + \frac{\pi}{3}\right)$ ∀ x∊ R and g:R⟶R be such that g$\left(\frac{5}{4}\right)$=1. Then prove that gof is a constant function.
53. Show that f:R⟶R defined by f(x)= $\frac{x}{1+x^2}$ is neither 1-1 nor onto.
54. Write all one-one functions on {a, b, c}
55. Let A = [0,1], if f: A⟶A is given by f(x) = $\begin{cases} x, & x \in Q \\ 1-x, & x \notin Q \end{cases}$ prove that fof=I_A
56. Let ∗ be a binary operation on Z defined by a∗b = a+b−15 ∀ a,b∈Z. check commutativity and associativity. Prove that identity is 15 and a^{-1} = 30 –a.
57. If f(x + y, x − y) = xy prove that f(x , y) = $\frac{x^2-y^2}{4}$
58. If f(x) = $\log_{x^2} x$, x∈ R^+−{1} , prove that f is many-one.
59. Show that sum of two one-one functions may not be one-one.
60. Let f(x) =[x] and g(x) =|x|, evaluate fog(1/3) – gof(−1/3).