DON BOSCO SCHOOL ALAKNANDA

CLASS XII (2017-18) **MATHEMATICS ASSIGNMENT : 2**

<u>CHAPTER: CONTINUITY AND DIFFERENTIABILITY</u>

1. Find the points of discontinuity of the following functions:

   (i) $\sin x \cos x$ (ii) $|x+1| + |x-1|$ (iii) $[x],\ x \in [3,8]$ (iv) $\{x\} = x - [x],\ x \in R$

   (v) $\frac{1}{x+2}$ (vi) $\frac{1}{\log x}$ (vii) $\frac{\log x}{\sqrt{1-4x^2}}$ (viii) $\frac{|x|}{x}$

2. If $f(x) = x^2 g(x)$, $g(1) = 6$ and $g'(x) = 3$; find the value of $f'(1)$.

3. Examine the continuity of the functions at the indicated points:

   (i) $f(x) = \begin{cases} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ at $x = 0$

   (ii) $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$

   (iii) $f(x) = \begin{cases} \frac{x-|x|}{2} & x \neq 0 \\ 2 & x = 0 \end{cases}$ at $x = 0$

   (iv) $f(x) = \begin{cases} \frac{e^x-1}{\log(1+2x)} & x \neq 0 \\ 4 & x = 0 \end{cases}$ at $x = 0$

   (v) $f(x) = \begin{cases} \frac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)} & x \neq \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & x = \frac{1}{\sqrt{2}} \end{cases}$ at $x = \frac{1}{\sqrt{2}}$

   (vi) $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$

   (vii) $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2} & x \neq 0 \\ \frac{b^2-a^2}{2} & x = 0 \end{cases}$ at $x = 0$

   (viii) $f(x) = \begin{cases} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & x \neq 0 \\ 4 & x = 0 \end{cases}$ at $x = 0$

4. A bike rider planning to take the path whose equation is given by $f(x) = \begin{cases} \frac{1-x^4}{1-x} & x \neq 1 \\ 1 & x = 1 \end{cases}$.

   Find the point in his path which he should not travel. Give reasons.

5. For what values of constant K, the following functions are continuous at the indicated points;

   (i) $f(x) = \begin{cases} \frac{e^x-1}{\log(1+2x)} & x \neq 0 \\ k & x = 0 \end{cases}$ at x=0

   (ii) $f(x) = \begin{cases} [x] + [-x] & x \neq 0 \\ k & x = 0 \end{cases}$ at x=0

   (iii) $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & x < 0 \\ \frac{2x+1}{x-1} & x \geq 0 \end{cases}$ at x=0

   (iv) $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$ at x=0

   (v) $f(x) = \begin{cases} k \sin\left(\frac{\pi(x+1)}{2}\right) & x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & x > 0 \end{cases}$ at x=0

   (vi) $f(x) = \begin{cases} \frac{\sin 3x}{\tan 2x} & x < 0 \\ k & x = 0 \\ \frac{\log(1+2x)}{e^{3x}-1} & x > 0 \end{cases}$ at x=0

   (vii) $f(x) = \begin{cases} \frac{x}{|x|+2x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ at x = 0

   (viii) $f(x) = \begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2} & x \neq 2 \\ k & x = 2 \end{cases}$ at x = 2

6. Find the values of a and b (or the value of k), if f(x) is continuous:

   (i) $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x} & x < 0 \\ 2 & x = 0 \\ \frac{\sqrt{1+bx}-1}{x} & x > 0 \end{cases}$

   (ii) $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & x < \frac{\pi}{2} \\ a & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & x > \frac{\pi}{2} \end{cases}$

   (iii) $f(x) = \begin{cases} \frac{x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2-4b}{x^2} & \sqrt{2} \leq x < \infty \end{cases}$

   (iv) $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & x < 0 \\ b & x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & x > 0 \end{cases}$

(v) $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a & x<4\\ a+b & x=4\\ \frac{x-4}{|x-4|}+b & x>4\end{cases}$ (vi) $f(x)=\begin{cases}a\sqrt{2}\sin x+x & 0\le x\le\frac{\pi}{4}\\ b+2x\cot x & \frac{\pi}{4}<x<\frac{\pi}{2}\\ a\cos 2x-b\sin x & \frac{\pi}{2}\le x\le\pi\end{cases}$

7. If $f(x)=\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ for $x\neq\frac{\pi}{4}$, find the value of $f\left(\frac{\pi}{4}\right)$ so that f(x) is continuous in $\left[0,\frac{\pi}{2}\right]$

8. Find the value(s) of a and b (or the value(s) of k), if f(x) is differentiable at the indicated point/everywhere:

(i) $f(x)=\begin{cases}x^2+3x+a & x\le 1\\ bx+2 & x>1\end{cases}$ at x = 1 (ii) $f(x)=\begin{cases}x^2 & x\le c\\ ax+b & x>c\end{cases}$ at x = c

(iii) $f(x)=\begin{cases}\frac{1}{|x|} & |x|\ge 1\\ ax^2+b & |x|<1\end{cases}$ (iv) $f(x)=\begin{cases}ax^2+1 & x>1\\ x+\ 1/2 & x\le 1\end{cases}$

(v) $f(x)=\begin{cases}x^k\sin(1/x) & x\neq 0\\ 0 & x=0\end{cases}$ at x=0

9. (a) If f(x) is differentiable at x=a, evaluate $\lim_{x\to a}\left(\frac{x^2 f(a)\ -\ a^2 f(x)}{x-a}\right)$

(b) If f(2) = 4 and f'(2) =1, then find the value of $\lim_{x\to 2}\left(\frac{x f(2)\ -2 f(x)}{x-2}\right)$

(c) If f(x) = λx^2+7x-4 and f'(5)= 97, find the value of λ

(d) Give an example of a function which is everywhere continuous but not differentiable at

(i) exactly one point (ii) exactly two points (iii) exactly 5 points

10. Find $\frac{dy}{dx}$:

(i) $y=\cos(\cot^{-1}x+\tan^{-1}x)$ (ii) y =3sinx° – 2 sinx (iii) $y=\log_{\sqrt{e}}\cos x$

(iv) $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ (v) $y=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ (vi) $y=sin^{-1}\left(\frac{x+x^{-1}}{x-x^{-1}}\right)$

(vii) $y=|\sin x|$ (viii) $y=(\sin x)^{\cos^{-1}x}$ (ix) $y=10^{10^x}$

(x) $y=\frac{\sqrt{x}\,(x+4)^{3/2}}{(4x-3)^{4/3}}$ (xi) $y=2e^{\sin^2x}\ .\ \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ (xii) $y=\frac{8^x}{x^8}$

(xiii) $y=\left(\sqrt{x}\right)^{\sqrt{x}}$ (xiv) $y=x^a+x^e+e^x+a^x+x^x+e^a+a^e+e^e$ (xv) $y=x^{\tan x}+\sqrt{\frac{x^2+1}{2}}$

(xvi) $y=\sin^{-1}\left(x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right)$ (xvii) $y=\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ (xviii) $y=\sqrt{1+\sqrt{1+x^4}}$

(xix) $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=4$ (xx) (xxi)

11. Differentiate the following w.r.t x and find $\frac{dy}{dx}$:

(i) $(\cos x)^y=(\cos y)^x$ *(ii)* $x^m y^n=(x+y)^{(m+n)}$ *(iii)* $y=\cos x^{\log x}+\log x^x+x^{\sqrt{x}}$

(iv) $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$ *(v)* $y=xe^y$ *(vi)* $(x^2-y^2)^2=4xy$

(vii) $y=\sqrt{x+\sqrt{x+\cdots\infty}}$ (viii) $y=(\cos x)^{(\cos x)^{\cdots\infty}}$ (ix) $y=\sqrt{\sin x+y}$

(x) $x=e^{\theta}\left(\theta+\frac{1}{\theta}\right),\ y=e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$ (xi) $x=t+\frac{1}{t}\quad y=t-\frac{1}{t}$ (xii) $x\,e^{xy}=y+\sin^2x$ at x=0

(xiii) $y=\log_{10}\sin x$ (xiv) $y=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$ (xv) $x=a\left[\cos t+\frac{\log\tan^2\left(\frac{t}{2}\right)}{2}\right], y=a\sin t$

12. (i) If $x=\frac{a}{1+m^3}$, $y=\frac{am}{1+m^3}$ prove that $\frac{dy}{dx}=\frac{2m^3-1}{3m^2}$

(ii) If $y=\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, prove that $xy_1=\sqrt{x^2+1}$

(iii) If $y=\sqrt{\frac{1+e^x}{1-e^x}}$ prove that $\frac{dy}{dx}=\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}=\frac{e^x\,y}{1-e^{2x}}$

(iv) If $x=\frac{1+\log t}{t^2}$, $y=\frac{3+2\log t}{t}$, t>0, show that $y\,y_1-2x\,(y_1)^2=1$

(v) If $y=\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$, prove that $\frac{dy}{dx}=\frac{5}{1+25x^2}-\frac{3}{1+9x^2}$

(vi) If $x=e^{\cos 2\theta}$ and $y=e^{\sin 2\theta}$, prove that $x\,y_1\log y+y\log x=0$

(vii) If $y=2\tan^{-1}\sqrt{\frac{x-a}{b-x}}$ and a<x<b, show that $\left(\frac{dy}{dx}\right)^2+\frac{1}{(x-a)(x-b)}=0$

(viii) If $y = e^x + e^{-x}$, then show that ${y_1}^2 + 4 = y^2$

(ix) If $y = (x-1)\log(x-1) - (x+1)\log(x+1)$, prove that $\frac{dy}{dx} = \log\left(\frac{x-1}{x+1}\right)$

(x) If $y = \log(\sqrt{x-1} - \sqrt{x+1})$, show that $\frac{dy}{dx} = -\frac{1}{2\sqrt{x^2-1}}$

(xi) If $y\sqrt{1+x^2} = \log(\sqrt{1+x^2} - x)$, prove that $(x^2+1)y_1 + xy + 1 = 0$

13. (a) If $x = \sec t - \cos t,\ y = \sec^4 t - \cos^4 t$, prove that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = 16(y^2+4)$

(b) If $x = \sec t - \cos t,\ y = \sec^n t - \cos^n t$, prove that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

(c) If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, prove that $x^3\, y\, y_1 = 1$

(d) Prove that derivative of an even function is an odd function and that of an odd function is an even function.

14. (a) If f′(0) = 2 g′(0), prove that derivative of $f(\sin^{-1}x)$ w.r.t. $g(x^2 + x)$ at x = 0 is equal to 2.

(b) if u = f(tanx), v = g(secx) and $f'(x) = \tan^{-1}x$, $g'(x) = \sec^{-1}x$; prove that $\left.\frac{du}{dv}\right|_{x=\frac{\pi}{4}} = \sqrt{2}$.

(c) If f(x + y)= f(x) f(y) ∀ x,y ∈ R and f(5) = 2, f′(0) = 3; show that f′(5) = 6.

15. (a) If f(1)=4 and f′(1)=2,find the derivative of $\log(f(e^x))$ w.r.t. x at x=0.

(b) Find $\frac{dy}{dx}$ if $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$

16. (i) If $x = \sin t, y = \sin mt$; show that $(1-x^2)y_2 - xy_1 + m^2y = 0$

(ii) If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $xy\, y_2 + x\, {y_1}^2 = y\, y_1$

(iii) If $y = \left(x + \sqrt{x^2+1}\right)^m$, prove that $(x^2+1)y_2 + xy_1 = m^2y$

(iv) If $y^{1/n} + y^{-1/n} = 2x$, Write y explicitly as function of x. And hence prove $(x^2-1)y_2 + xy_1 = n^2y$

(v) If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, show that $y^2y_2 - xy_1 + y = 0$

(vi) If $x = \sin\left(\frac{\log y}{a}\right)$, prove that $(1-x^2)y_2 - xy_1 = a^2y$

(vii) If $y = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$, show that $y_2 = \frac{b\sin x}{(a+b\cos x)^2}$

(viii) If $y = \sqrt{x-1} - \sqrt{x+1}$, prove that $(x^2-1)y_2 + xy_1 = \frac{y}{4}$

(ix) If $y = \log(1+\cos x)$, prove that $y_3 + y_2\, y_1 = 0$

(x) If $y = (\tan^{-1}x)^2$, prove that $(x^2+1)^2\, y_2 + 2x(x^2+1)y_1 = 2$

(xi) If $x^p y^q = (x+y)^{(p+q)}$, prove that $y_1 = \frac{y}{x}$ and $y_2 = 0$

(xii) If $y = \sin(2\sin^{-1}x)$, prove that $(1-x^2)y_2 - xy_1 + 4y = 0$

(xiii) If $x = (a+bt)\,e^{-nt}$, prove that $\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + n^2x = 0$

(xiv) If $y = \sec x + \tan x$, show that $x\, y_2 = y^2$

(xv) If $y = e^x\tan^{-1}x$, $(x^2+1)y_2 - 2(1-x+x^2)y_1 + (1-x)^2y = 0$

17. (a) If $f(x) = \tan^{-1}x$ and $g(x) = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, |x|<1, prove that $\frac{df}{dg} = 1$

(b) Find the slope of normal and tangent to the curve $f(x) = \frac{\log x}{x}$ at x = 1.

(c) Find the rate at which the function $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ changes w.r.t. the function $\sin^{-1}(2x\sqrt{1-x^2})$

(d) Find the derivative of $\log(1+x^2)$ w.r.t. $\cot^{-1}x$

18. (i) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

(ii) If $x\sqrt{1-y^2} \pm y\sqrt{1-x^2} = a$, prove that $\frac{dy}{dx} = \mp\sqrt{\frac{1-y^2}{1-x^2}}$

(iii) If $\sqrt{1-x^2} \pm \sqrt{1-y^2} = a(x \mp y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(iv) If $\sqrt{1-x^2} \pm \sqrt{1-y^2} = a(x \pm y)$, prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

(v) If $\sqrt{1-x^6} \pm \sqrt{1-y^6} = a(x^3 \mp y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

(vi) If $\sqrt{1-x^6} \pm \sqrt{1-y^6} = a(x^3 \pm y^3)$, prove that $\frac{dy}{dx} = -\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

In General, prove the following: (vii) and (viii)

(vii) If $\sqrt{1-x^{2n}} \pm \sqrt{1-y^{2n}} = a(x^n \mp y^n)$, prove that $\frac{dy}{dx} = \left(\frac{x}{y}\right)^{n-1}\sqrt{\frac{1-y^{2n}}{1-x^{2n}}}$

(viii) If $\sqrt{1-x^{2n}} \pm \sqrt{1-y^{2n}} = a(x^n \pm y^n)$, prove that $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1}\sqrt{\frac{1-y^{2n}}{1-x^{2n}}}$

(ix) If $\sqrt{1+x^2} \pm \sqrt{1+y^2} = a(x \mp y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$

(x) If $\sqrt{1+x^2} \pm \sqrt{1+y^2} = a(x \pm y)$, prove that $\frac{dy}{dx} = -\sqrt{\frac{1+y^2}{1+x^2}}$

19. (i) Discuss the applicability of Rolle's Theorem:

(a) f(x) = $3 + (x-2)^{2/3}$ on [1, 3] (b) f(x) = tanx on $[0, \pi]$ (c) f(x) = $\begin{cases} x^2+1 & 0 \le x \le 1 \\ 3-x & 1 < 1 \le 2 \end{cases}$

(ii) Verify Rolle's Theorem :

(a) f(x)= $x^3 - 7x^2 + 16x - 12$ 0n [2, 3] (b) f(x)=log(x^2 + 2) – log3 on [–1, 1] (c) f(x)=e^x(sinx – cosx) on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

(iii) Using Rolle's Theorem, find the point(s) on the curve $y = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$ where tangent is parallel to X axis

(iv) Using Rolle's Theorem, find the point(s) on the curve $y = x^2 + 2$ between the points corresponding to y=3, where tangent is parallel to X axis.

20. (i) Use LMVT to find point(s) on the curve $y = \sqrt{x^2 - 4}$ in [2, 4, where the tangent is parallel to the chord joining the end points on the curve.

(ii) Verify the Hypothesis and Conclusion of LMV Theorem for f(x) = $\frac{1}{4x-1}$ $1 \le x \le 4$

(iii) Using LMVT find point(s) on the curve f(x) = x^2 + x where tangent is parallel to the chord joining the origin and the point(1,2)

ANSWERS:

1. (i) ∅s (ii) ∅ (iii) 4,5,6,7 (iv) $x \in Z$ (v) $cont\ in\ D_f = R - \{-2\}$ (vi) $cont\ inD_f = (0, \infty) - \{1\}$ (vii) $cont\ inD_f = \left(0, \frac{1}{2}\right)$ (viii) $cont\ in\ D_f = R - \{0\}$
2. 15
3. (i) Continuous (ii) Continuous (iii) Discontinuous (iv) Discontinuous (v) Continuous (vi)Discontinuous (vii) Continuous (viii) Discontinuous
4. At x =1 as f(x) is discontinuous at x= 1.
5. (i) k = $\frac{1}{2}$ (ii) k = – 1 (iii) k = – 1 (iv) k = 8 (v) k = 1/2 (vi) no value of k (vii) no value of k (viii) k = 7
6. (i) a = – 1, b = 4 (ii) $a =$ 1/2, b = 4 (iii) a = – 1, 1 b = 1, $1 \pm \sqrt{2}$ (iv) $a = -\frac{3}{2}, b = \frac{1}{2}$ (v) a =1, b = – 1 (vi) $a = \frac{\pi}{6}$ $b = -\frac{\pi}{12}$
7. 1/2
8. (i) a = 3, b= 5 (ii) a = 2c, b= – c^2 (iii) a=– 1/2, b= 3/2 (iv) a = 1/2 (v) k >1
9. (a) 2a f(a) – a^2 f'(a) (b) 2 (c) $\lambda = 9$
10. (i) 0 (ii) $3\frac{\pi}{180}\cos x^{o}$ – 2 cosx (iii) – 2tanx (iv) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (v) $\frac{2^{x+1}}{1+4^x}\log 2$ (vi) $\frac{2}{1+x^2}$ (vii) y cotx
(viii) $(\sin x)^{\cos^{-1}x}\left[\cos^{-1}x\ \cot x - \frac{\log(\sin x)}{\sqrt{1-x^2}}\right]$ (ix) $10^{10^x}10^x(\log 10)^2$ (x) $\frac{\sqrt{x}\,(x+4)^{3/2}}{(4x-3)^{4/3}}\left[\frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)}\right]$
(xi) $e^{\sin^2 x}\left[\sin 2x.\cos^{-1}x - \frac{1}{\sqrt{1-x^2}}\right]$ (xii) $\frac{8^x}{x^8}\left(\log 8 - \frac{8}{x}\right)$ (xiii) $\frac{2+\log x}{4}\ \sqrt{x}^{(\sqrt{x}-1)}$ (xiv) $ax^{a-1} + ex^{e-1} + e^x + x^x(1+\log x) + a^x\log a$
(xv) $x^{(\tan x-1)}[x\log x\sec^2 x + \tan x] + \frac{x}{\sqrt{2(1+x^2)}}$ (xvi) $\frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$ (xvii) 0 (xviii) $\frac{x^3}{y(y^2-1)}$ (xix) $\frac{x-7y}{7x-y}$
11. (i) $\frac{\log(\cos y)+y\tan x}{\log(\cos x)+x\tan y}$ (ii) $\frac{y}{x}$ (iii) $x^{-1}\cos x^{\log x}[\log(\cos x) - \tan x\log x] + \log x^x[\log(\log x) + (\log a)^{-1}] + x^{(\sqrt{x}-1/2)}\left[1 + (\frac{\log x}{2})\right]$ (iv) $\frac{y}{x}$
(v) $\frac{y}{x(1-y)}$ (vi) $\frac{-x^3+xy^2+y}{y^3-x^2y-x}$ (vii) $\frac{1}{2y-1}$ (viii) $\frac{y^2\tan x}{y\log(\cos x) - 1}$ (ix) $\frac{\cos x}{2y-1}$ (x) $\frac{1+\theta+\theta^2-\theta^3}{e^{2\theta}(\theta^3+\theta^2+\theta-1)}$ (xi) $\frac{t^2+1}{t^2-1}$ (xii) 1 (xiii) $\frac{\cot x}{\log 10}$
(xiv) $\sqrt{a^2 - x^2}$ (xv) tan t

15. (a) 1/2 (b) $2\sin\left(\frac{2x-1}{x^2+1}\right)^2.\ \left[\frac{1+x-x^2}{(x^2+1)^2}\right]$
17. (b) – 1 and 1 (c) 1/2 (d) – 2x (e)